Application of a Simplex Method to Find the Optimal Solution

M.V.Sarode

Dept. of Mathematics, Priyadarshini Indira Gandhi College of Engineering, Nagpur.

Abstract – *In this paper we discuss the graphical and simplex method to maximize the profit of chair maker with his limited constraints(resources). Here we find the solution with both the method to solve a class of linear programming problem having all constraints $\leq$ type.*

Keywords- *Linear programming problem, Simplex method, Graphical method, Profit.*

INTRODUCTION

Linear programming model is extensively used to solve the variety of problems in Engineering and Management field particularly for production planning and optimum resources allocation. To solve linear programming problem, Dantzig devised simplex method in 1947. Asimplex algorithm of Dantzig is one of the top 10 algorithms of 20 th century [Barry Cipra (3)]. The computational complexity in simplex method depends on the number of variables and the number of constraints and is directly proportional to both. To reduce the complexity, several variants of simplex method are developed by various researchers. For details Gass (2) can be referred. If in the given linear programming problem, one or more constraints are $\geq$ type or equality, simplex method with artificial basis techniques is used. Big-M method and two phase simplex method are quite commonly used. In these methods artificial variables are used to get the standard basis artificially. Artificial variables are then forced to leave the basis step by step. Once all the artificial variables are removed from the basis, optimal solution is then obtained by using regular simplex method. However, use of artificial variables to get the standard basis increases the computational complexity due to the reason mentioned above. To avoid the use of artificial variables, the $\geq$ type constraints are converted into $\leq$ type constraints by multiplying by -1. Then using slack variables only,

initial basic solution is obtained that is not feasible. In other words, one starts with infeasible solution. Methods are developed to solve linear programming problem by starting with infeasible solution and force the solution to be feasible as well as optimal at some iteration. One such method namely dual simplex method devised by Lemke (5) is most popular. In dual simplex method, starting from infeasible solution feasible solution is obtained step by step. Another basis-exchange pivoting algorithm is the criss-cross algorithm by Terlaky, Tamás (4) .There are polynomial-time algorithms for linear programming that use interior point methods: These include Khachiyan's ellipsoidal algorithm, Karmarkar's projective algorithm, and path-following algorithms. Interested readers can refer Robert J. Vanderbeib (1). In the proposed method in this article, we start with initial basic solution that is infeasible. Then by judicious selection of entering and leaving vector in and from the initial simplex table, feasible solution is obtained in the first iteration only and optimal solution then can be obtained using regular simplex method. Notations: The proposed method can be used to solve the linear programming problem of the type furniture manufacture-turer wishes to maximize profit. Information about available resources (board feet of wood and hours of labor) and the object-tive criterion is presented in Table1.

Table 1

Resource	Table (X_1)	Chair (X_2)	Available
Wood (bf)	30	20	300
Labor (hr)	5	10	110
Unit profit	Rs.6	Rs.8	

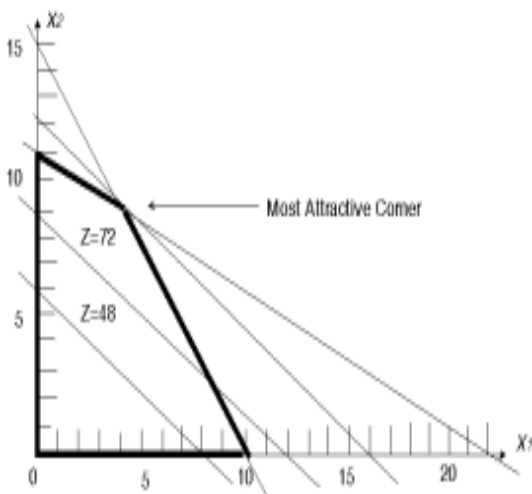Maximize: $Z = 6 X_1 + 8X_2$ (objective function)

Subject to: $30 X_1 + 20 X_2 < 300$ (wood constraint: 300 bf available)

$5 X_1 + 10 X_2 < 110$ (labor constraint: 110 hours available)

X_1 , $X_2 > 0$ (nonnegative conditions)

Based on the above information, graphically solve the LP (Figure 1). Graph the two constraint equation lines. Then plot two objective function lines by arbitrarily setting $Z = 48$ and $Z = 72$ to find the direction to move to determine the most attractive corner. The coordinates for the most attractive corner (where the wood and labor constraint equations intersect) can be found by simulta-

neously solving the constraint equations with two unknowns.

To simultaneously solve the two constraint equations, first multiply the labor equation by -2, and add it to the wood equation:

$30 X_1 + 20X_2 = 300$ (wood)

$-2 (5 X_1 + 10X_2 = 110)$ (labor)

$20X_1 + 0X_1 = 80 = 4$ tables

Next, substitute into either of the constraint equations to find the number of chairs. We can substitute into both equations to illustrate that the same value is found. Wood constraint Labor constraint

$30(4) + 20X_2 = 300 \quad 5(4) + 10X_2 = 110$

$120 + 20X_2 = 300 \quad 20 + 10X_2 = 110$

$20X_2 = 300 - 120 \quad 10X_2 = 110 - 20$

$20X_2 = 180 \quad 10_{X2} = 90$

$X_2 = 180/20 \quad X_2 = 90/10$

$X_2 = 9$ chairs $X_2 = 9$ chairs

Now, determine the value of the objective function for the optimal solution. Substitute into the equation the number of tables and chairs, and solve for Z.

$Z = Rs.6(4) + Rs.(9) = Rs.96$

The optimal solution is to manufacture four tables and nine chairs for a profit of Rs.96.

Using the simplex method

By introducing the idea of slack variables (unused resources) to the tables and chairs problem, we can add two more variables to the problem. With four variables, we can't solve the LP problem graphically. We'll need to use the simplex method to solve this more complex problem. We'll briefly present the steps involved in using the simplex method before working through an example. Table 2 shows an example of a simplex tableau. Although these steps will give you a general overview of the procedure, you'll probably find that they become much more understandable as you work through the example. Using the simplex method, the first step is to recognize surplus resources, represented in the problem as slack variables. In most real-life problems, it's unlikely that all resources (usually a large mix of many different resources) will be used completely. While some might be used completely, others will have some unused capacity. Also, slack variables allow us to change the inequalities in the constraint equations to equalities, which are easier to solve algebraically. Slack variables represent the unused resources between the left-hand side and right-hand side of each inequality; in other words, they allow us to put the LP problem into the stan- dard form so it can be solved using the simplex method. The first step is to convert the inequalities into equalities by adding slack variables to the two constraint inequalities. With S W representing surplus wood, and S L representing surplus labor, the constraint equations can be written as:

$30X_1 + 20X_2 + S_W = 300$ (wood constraint: 300 bf)
$5X_1 + 10X_2 + S_L = 110$ (labor constraint: 110 hours)

All variables need to be represented in all equations. Add slack variables to the other equations and give them

coefficients of 0 in those equations. Rewrite the objective function and constraint equations as:

Maximize: $Z = 6 X_1 + 8X_2 + 0S_W + 0 S_L$ (objective function)

Subject to: $30 X_1 + 20X_2 + S_W + 0 S_L = 300$ (wood constraint: 300 bf)

$5X_1 + 10X_2 + 0S_W + S_L = 110$ (labor constraint: 110 hours)

$X_1 , X_2, S_W , S_L > 0$ (non negativity conditions)

We can think of slack or surplus as unused resources that don't add any value to the objective function. Thus, their coefficients are 0 in the objective function equation.

Unit Profit

Basic mix	6	8	0	0	
	X_1	X_2	S_W	S_L	Solution
S_W	30	20	1	0	300
S_L	5	10	0	1	110

The next step is to determine the exiting variable. The exiting variable is the variable that will exit the basic mix when you construct your next simplex tableau. We'll find the exiting variable by calculating the exchange ratio for each basic variable. The exchange ratio tells us how many tables or chairs can be made by using all of the resource for the current respective basic variable. To find the exchange ratio, divide the solution value by the corresponding exchange coefficient in the entering variable column.

The exchange ratios are:
$300/20 = 15$ (S_W basic mix row)
and $110/10 = 11$ (S L basic mix row)

By using all 300 board feet of wood, we can make 15 chairs because it takes 20 board feet of wood to make a chair. By using all 110 hours of labor, we can make 11 chairs because it takes 10 hours of labor per chair. Thus, it's easy to see the plant can't manufacture 15 chairs. We have enough wood for 15 chairs but only enough labor for 11.

Unit Profit

	Basic mix	6	8	0	0		
		X_1	$X_2 \downarrow$	S_W	S_L	Solution	
0	S_W	30	20	1	0	300	300/20 =15
0	$S_L \leftarrow$	5	10	0	1	110	110/10 =11
	Sacrifice	0	0	0	0	0 ←	Current profit
	Improve ment	6	8	0	0	--	

Now we'll find the values for the S W row. Referring to Table 8a, find the value in the S W row in the old tableau in the pivot element column (20). Multiply it times the first value in the new X 2 row (0.5 from Table 8b). Subtract your answer from the value in the first position of the old S W row.

Unit Profit

	Basic mix	6	8	0	0		
		X_1	$X_2 \downarrow$	S_W	S_L	Solution	
0	S_W	20	0	1	-2	80	300/20 =15
8	$S_L \leftarrow$	0.5	1	0	0.1	11	110/10 =11
	Sacrifice	4	8	0	0.8	88 ←	Current profit
	Improve ment	2	0	0	-0.8	--	

We now see that profit has been improved from 0 to Rs. 88. Replace the entering variable in the basic mix where the exiting variable left. Bring over the unit profit from the top row of the old table to the new table. Fill in the pivot element row by dividing through by the pivot element. The greatest per-unit improvement is 2 (X $_1$ column). The others offer no improvement (either 0 or a negative number). X_1 becomes the new entering variable. Mark the top of its column with an arrow (Table 8e). Remember, when no improvement can be found at this step, the current tableau represents the optimal solution. Now determine the exiting variable. To do so, first determine the exchange ratios:

$80/20 = 4$

and $11/0.5 = 22$

Now choose the

Unit Profit

	Basic mix	6	8	0	0		
		$X_1 \downarrow$	X_2	S_w	S_L	Solution	
0	$S_W \leftarrow$	20	0	1	-2	80	80/20=4
8	X_2	0.5	1	0	0.1	11	11/0.5=22
	Sacrifice	4	8	0	0.8	88 $\leftarrow$	
	Improvement	2	0	0	-0.8	--	

Unit Profit

	Basic mix	6	8	0	0	
		X_1	$X_2 \downarrow$	S_W	S_L	Solution
6	X_1	1	0	0.05	-0.1	4
8	$X_2 \leftarrow$	0	1	-0.025	0.1	9
	Sacrifice	6	8	0.1	0.6	96
	Improvement	0	0	-0.1	-0.6	--

There are no positive numbers in the new improvement row. Thus, we no longer can improve the solution to the problem. This simplex tableau represents the optimal solution to the LP problem
and is interpreted as: $X_1 = 4$, $X_2 = 9$, $S_W = 0$, $S_L = 0$, and profit or $Z = Rs.96$
The optimal solution (maximum profit to be made) is to manufacture four tables and nine chairs for a profit of Rs.96.

CONCLISION

Thus here we discuss two methods to find optimal solution. Here we find out that there is restriction to graphical method that which is applies only two objective variables whereas simplex methods apply two or more objective variable.

REFERENCES

[1] *Jervin Zen Lobo Department of Mathematics, St Xavier's College, Mapusa, Goa, India.'Two Square Determinant Approach for Simplex Method'. IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 5 Ver. IV (Sep. - Oct. 2015), PP 01-04.*

[2] *Md. Mijanoor Rahman, Md. Kamruzzaman Department of Civil Engineering, Presidency University, Gulshan-2, Dhaka, Bangladesh' NETWORK SIMPLEX METHOD USED TO TRANSPORTATION PROBLEM'American International Journal of Research in Science, Technology, Engineering & MathematicsAvailable online at http://www.iasir.net ISSN (Print): 2328-3491, ISSN (Online).*

[3] *Dr. R.G. Kedia Assistant Professor & Head Department of Statistics, Govt. Vidarbha Institute of Science & Humanities, Amravati, INDIA 'A New Variant of Simplex Method' nternational Journal of Engineering and Management Research Volume-3, Issue-6, December-2013, ISSN No.: 2250-0758.*

[4] *H. Hashamdar* Z. Ibrahim, M. Jameel, A. Karbakhsh , Z. Ismail and M. Kobraei'Use of the simplex method to optimize analytical condition in structural analysis' International Journal of Physical Sciences Vol. 6(4), pp. 691-697, 18 February, 2011 Available online at http://www.academicjournals.org/IJPS ISSN 1992 -1950 ©2011 Academic Journals*

[5] *Kirtiwant P. Ghadle,Tanaji S. Pawar NEW APPROACH FOR WOLFE'S MODIFIED SIMPLEX METHOD TO SOLVE QUADRATIC PROGRAMMING PROBLEMS IJRET: International Journal of Research in Engineering and Technology eISSN: 2319-1163 | pISSN: 2321-7308*