Impact Factor Value 3.441 e-ISSN: 2456-3463
International Journal of Innovations in Engineering and Science, Vol. 2, No.7, 2017
www.ijies.net

Application of a Dual Simplex method to Transportation Problem to minimize the cost

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Abstract – *In this paper the aim of work is to introduce dual simplex method to solve transportation problem with fuzzy objective functions. The Simplex has wide use transportation problem. It aims to minimize the cost of transportation. The fuzzy objective functions have fuzzy demand and supply coefficients, which are represented as fuzzy numbers. For this we are solving fuzzy transportation problem by dual simplex method.*

Keywords - Linear programming problem, Simplex method, Transportation problem, Profit.

1. INTRODUCTION

The transportation problem is a special type of linear programming problem which arises in many practical applications. The transportation model has wide practical applications, not only in transportation systems, but also in other systems such as production planning [3]. The concept of fuzzy set theory, first introduced by Zadeh [19] is used for solving different types of linear programming problems [1]. After that, especially after 1990s various models and algorithms under both crisp environment and uncertain environment are presented. For ex. Bit Et [2]. Al, Jimnez and Verdegay [7,8], Li et. Al. [9], Srinivasan and

Thompsom [18], L. Liu and B. Liu [12]. Liu and Kao [13] developed a method to find the membership function of the fuzzy total transportation cost when the unit shipping costs, the supply quantities, and the demand quantities are fuzzy numbers. Jimenez and Verdegay [8] proposed a GA to deal with the fuzzy solid transportation problem in which the fuzziness affects only in the constraint set. They concluded that GA showed a good performance in finding parametric solutions in comparison with nonparametric solutions obtained with other nonlinear solution methods. Lin and Tsai [11] investigated solving the transportation problem with fuzzy demands and fuzzy supplies using a two-stage GA. This study suggests methods of linear programming approach to solving the transportation problem with fuzzy demands and fuzzy supplies. The numerical solved by dual simplex method. In this method the coefficients of objective function are in the form of fuzzy numbers and changing problem in linear programming problem then solved by dual simplex method. At first Danting G.A. and P. Wolfe [5] (1955) generalised simplex method for minimizing a linear form under inequality restraints. The simplex method starts with a dictionary which is feasible but does not satisfy the optimality condition on the Z equation. It performs successive pivot operations preserving feasibility to find a dictionary which is both feasible and optimal. The dual simplex method starts with a dictionary which satisfies the optimality condition on the Z equation, but is not feasible. It then performs successive pivot operations, which preserve optimality, to find a dictionary which is both feasible and optimal.

Linear programming model is extensively used to solve the variety of problems in Engineering and

Management field particularly for production planning and optimum resources allocation. To solve linear programming problem, Dantzig devised simplex method in 1947. ASimplex algorithm of Dantzig is one of the top 10 algorithms of 20 th century [Barry Cipra (3)]. The computational complexity in simplex method depends on the number of variables and the number of constraints and is directly proportional to both. To reduce the complexity, several variants of simplex method are developed by various researchers. For details Gass (2) can be referred. If in the given linear programming problem, one or more constraints are ≥ type or equality, simplex method with artificial basis techniques is used.

Big-M method and two phase simplex method are quite commonly used. In these methods artificial variables are used to get the standard basis artificially. Artificial variables are then forced to leave the basis step by step. Once all the artificial variables are removed from the basis, optimal solution is then obtained by using regular simplex method. However, use of artificial variables to get the standard basis increases the computational complexity due to the reason mentioned above. To avoid the use of artificial variables, the ≥ type constraints are converted into ≤ type constraints by multiplying by -1. Then using slack variables only, initial basic solution is obtained that is not feasible. In other words, one starts with infeasible solution. Methods are developed to solve linear programming problem by starting with infeasible solution and force the solution to be feasible as well as optimal at some iteration. One such method namely dual simplex method devised by Lemke (5) is most popular. In dual simplex method, starting from infeasible solution feasible solution is obtained step by step. Another basis-exchange pivoting algorithm is the criss-cross algorithm by Terlaky, Tamás (4) .There are polynomial-time algorithms for linear programming that use interior point methods: These include Khachiyan's ellipsoidal algorithm, Karmarkar's projective algorithm, and path-following algorithms. Interested readers can refer Robert J. Vanderbeib (1). In the proposed method in this article, we start with initial basic solution that is infeasible. Then by judicious selection of entering and leaving vector in and from the initial simplex table, feasible solution is obtained in the first iteration only and optimal solution then can be obtained using regular simplex method. Notations: The proposed method can be used to solve the linear programming problem of the type

Using the simplex method

By introducing the idea of slack variables (unused resources) to the tables and chairs problem, we can add two more variables to the problem. With four variables, we can't solve the LP problem graphically. We'll need to use the simplex method to solve this more complex problem.

We'll briefly present the steps involved in using the simplex method before working through an example. Table 2 shows an example of a simplex tableau. Although these steps will give you a general overview of the procedure, you'll probably find that they become much more understandable as you work through the example.

Using the simplex method, the first step is to recognize surplus resources, represented in the problem as slack variables. In most real-life problems, it's unlikely that all resources (usually a large mix of many different resources) will be used completely. While some might be used completely, others will have some unused capacity. Also, slack variables allow us to change the inequalities in the constraint equations to equalities, which are easier to solve algebraically. Slack variables represent the unused resources between the left-hand side and right-hand side of each inequality; in other words, they allow us to put the LP problem into the stan- dard form so it can be solved using the simplex method. The first step is to convert the inequalities into equalities by adding slack variables to the two constraint inequalities. With S W representing surplus wood, and S L representing surplus labor, the constraint equations can be written as:

$30X_1 + 20X_2 + S_W = 300$ (wood constraint: 300 bf)

$5X_1 + 10X_2 + S_L = 110$ (labor constraint: 110 hours)

All variables need to be represented in all equations. Add slack variables to the other equations and give them coefficients of 0 in those equations. Rewrite the objective function and constraint equations as:

Maximize: $Z = 6X_1 + 8X_2 + 0S_W + 0\ S_L$(objective function)

Subject to: $30X_1 + 20X_2 + S_W + 0\ S_L = 300$ (wood constraint: 300 bf)

$5X_1 + 10X_2 + 0S_W + SL = 110$ (labor constraint: 110 hours)

X_1 , X_2, S_W , $S_L > 0$ (nonnegative conditions)

We can think of slack or surplus as unused resources that don't add any value to the objective function. Thus, their coefficients are 0 in the objective function equation.

Unit Profit

Basic mix	6	8	0	0	
	X_1	X_2	S_W	S_L	Solution
S_W	30	20	1	0	300
S_L	5	10	0	1	110

The next step is to determine the exiting variable. The exiting variable is the variable that will exit the basic mix when you construct your next simplex tableau. We'll find the exiting variable by calculating the exchange ratio for each basic variable. The exchange ratio tells us how many tables or chairs can be made by using all of the resource for the current respective basic variable. To find the exchange ratio, divide the solution value by the corresponding exchange coefficient in the entering variable column. The exchange ratios are:

300/20 = 15 ($S_W$basic mix row)

and 110/10 = 11 (S L basic mix row)

By using all 300 board feet of wood, we can make

15 chairs because it takes 20 board feet of wood to make a chair. By using all 110 hours of labor, we can make 11 chairs because it takes 10 hours of labor per chair. Thus, it's easy to see the plant can't manufacture 15 chairs. We have enough wood for 15 chairs but only enough labor for 11.

Unit Profit

	Basic mix	6	8	0	0		
		X_1	X_2↓	S_W	S_L	Solution	
0	S_W	30	20	1	0	300	300/20 =15
0	S_L←	5	10	0	1	110	110/10 =11
	Sacrifice	0	0	0	0	0 ←	Current profit
	Improvement	6	8	0	0	--	

Now we'll find the values for the S W row. Referring to Table 8a, find the value in the S W row in the old tableau in the pivot element column (20). Multiply it times the first value in the new X 2 row (0.5 from Table 8b). Subtract your answer from the value in the first position of the old S W row.

Unit Profit

	Basic mix	6	8	0	0		
		X_1	X_2↓	S_W	S_L	Solution	
0	S_W	20	0	1	-2	80	300/20 =15
8	S_L←	0.5	1	0	0.1	11	110/10 =11
	Sacrifice	4	8	0	0.8	88 ←	Current profit
	Improvement	2	0	0	-0.8	--	

We now see that profit has been improved from 0 to Rs. 88. Replace the entering variable in the basic mix where the exiting variable left. Bring over the unit profit from the top row of the oldtable to the new table. Fill in the pivot element row by dividing through by the pivot element.

The greatest per-unit improvement is2 (X_1 column). The others offer no improvement (either 0 or a negative number). X 1 becomes the new entering variable. Mark the top of its column with an arrow (Table 8e). Remember, when no improvement can be found at this step, the current tableau represents the optimal solution. Now determine the exiting variable. To do so, first determine the

exchange ratios:

80/20 = 4

and 11/0.5 = 22

Now choose the s

Unit Profit

	Basic mix	6	8	0	0		
		X_1 ↓	X_2	S_W	S_L	Solution	
0	S_W←	20	0	1	-2	80	80/20=4
8	x2	0.5	1	0	0.1	11	11/0.5=22
	Sacrifice	4	8	0	0.8	88 ←	
	Improvement	2	0	0	-0.8	--	

Unit Profit

	Basic mix	6	8	0	0	
		X_1	X_2↓	S_W	S_L	Solution
6	x1	1	0	0.05	-0.1	4
8	x2←	0	1	-0.025	0.1	9
	Sacrifice	6	8	0.1	0.6	96
	Improvement	0	0	-0.1	-0.6	--

There are no positive numbers in the new improvement row.Thus, we no longer can improve the solution to the problem. This simplex tableau represents the optimal solution to the LP problem

and is interpreted as: $X_1 = 4$, $X_2 = 9$, $S_W = 0$, $S_L = 0$, and profit or Z = Rs.96

The optimal solution (maximum profit to be made) is to manu-facture four tables and nine chairs for a profit of Rs.96.

The Simplex method for solving transportation problem:

A company has three toys factories located in cities A, B, C which supply toys to four projects in towns 1,2,3,4. Each plant can supply 6,1,10 truck loads of toys daily respectively and daily toys requirements of the projects are repectively 7,5,3,2 truck loads. The transportation costs per truck load of toys (in hundred Rs) from each plant to each project site are as follows:

	Project cities				
Factories		1	2	3	4
	1	2	3	11	7
	2	1	0	6	1
	3	5	8	15	9

Determine the optimal distribution for the company so as to minimize the total transportation cost.
First we construct the transportation table and we express the supply from the factories, demands at cities and the unit shipping cost in following table.

Factories	Project cities					
		1	2	3	4	Supply
	1	2	3	11	7	6
	2	1	0	6	1	1
	3	5	8	15	9	10
Demand		7	5	3	2	17

Now find the initial feasible solution. Transportation cost according to route is

$$Z = Rs.(1 \times 2 + 5 \times 3 + 1 \times 1 + 6 \times 5 + 3 \times 15 + 1 \times 9) times\ 100 = 10{,}200.$$

As the number of a allocation=m+n-1=6, thus we can apply MODI method.
Now we can compute net evolutions $w_{ij} = (u_i + v_j) - c_{ij}$

1	2	5	3	11	7
1	0	6	1	1
6	5	8	15	1	9

Now draw a closed path beginning and ending at (θ) cell. Add and substract (θ) *to and from transition cell.*

1	2	5	3	11		7	
1		0		θ	6	1	1(-θ)
6	5	8		3	15(-θ)	1	9(θ)

Transportation cost of this revised solution is

$$= Rs.(1 \times 2 + 5 \times 3 + 1 \times 6 + 6 \times 5 + 2 \times 15 + 2 \times 9) times\ 100 = Rs.10{,}100.$$

Since the cell (1,3) has a positive value, the second basic feasible solution is not optimal.

1	2	5	3	11(+)	7(-)	
1(-)		0(-)		6(+)	1	1
6	5	8(-)		15	1	9

1	2	5	3	11		7	
1		0		$\theta = 1$	6	1-1	1
6	5	8		3-1	15	1+1	9

1-1	2	5	3	11(θ = 1)		7	
	1		0	1	6	1	
6+1	5		8	2-1	15	2	9

1	2	5	3	11(+)		7(-)	
1(-)		0(-)		1	6	1 (-)	
6	5	8(-)		2	15	2	9

2(-)	5	3	1	11	7(-)	
1(-)		0(-)	1	6	1(-)	
7	5	8(-)	1	15	2	9

2(-)	5	3	1	11	7(-)	
1(-)		0(-)	1	6	1(-)	
7	5	8(-)	1	15	2	9

Since all the net evalutions are≤, this basic feasible solution is optimal.
Transportation cost of this revised solution is

$$= Rs.(5 \times 3 + 1 \times 11 + 1 \times 6 + 7 \times 5 + 1 \times 15 + 2 \times 9) times\ 100 = Rs.10{,}000.$$

CONCLISION

Thus here we discuss the simplex method to find optimal solution of transportation problem. The method of simplex is widely used to find the optimal solution. It applied to many areas of science, technology and management. It has been prove that simplex mehod is use minimize the cost of transportation with limitation of constraints. Transportation problem by general simplex method is possible but very time consuming where as especial methods like SSM & MODI solves the problem very easily but considering transportation problem as network flow problem and then solving it Network Simplex algorithm is more convenient.

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